Intermodal Dispersion

Let's look at the speed of a ray in the core material

$\beta = n_1 \frac{\omega}{c}$ $\quad n_1$ varies with ω

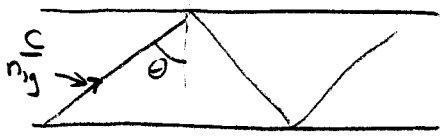
$$\frac{d\beta}{d\omega} = \frac{n_1}{c} + \frac{\omega}{c}\frac{dn_1}{d\omega} = \frac{1}{V_g}$$

$$\frac{1}{V_g} = \frac{1}{c}\left(n_1 + \omega \frac{dn_1}{d\omega}\right)$$

call this n_{1g} Group index of refraction

$$V_g = \frac{c}{n_{1g}}$$

Now look at the speed of a mode using a ray analysis



since the ray bounces, the z-speed is slower so

$$V_g = \frac{c}{n_{1g}} \sin\theta$$

If you look at the range of possible angles for a multimode waveguide.

$$\theta_c < \theta < 90°$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\frac{n_2}{n_1} < \sin\theta < 1$$

$$\frac{c}{n_{1g}}\frac{n_2}{n_1} < V_g < \frac{c}{n_{1g}}$$

The travel time is $T = \frac{L}{V_g}$

So pulse spread is $\Delta T = (T_1 - T_2) = L\left(\frac{1}{V_{g\min}} - \frac{1}{V_{g\max}}\right)$

$$\frac{\Delta T}{L} = \text{Dispersion} = \left(\frac{n_1}{n_2}\right)\left(\frac{n_{1g}}{c}\right) - \frac{n_{1g}}{c}$$

$$D_{inter} = \frac{n_{1g}}{c}\left(\frac{n_1}{n_2} - 1\right)$$

$$= \frac{n_{1g}}{c}\left(\frac{n_1}{n_2}\right)\left(\frac{n_1 - n_2}{n_1}\right)$$

$$D_{inter} \approx \frac{n_{1g}}{c}\Delta \quad \text{for highly multimode}$$

SMF28
$n_2 = 1.444$
$n_{1g} = 1.468$
$\Delta = 0.0036$

for SMF28 $D_{inter} = 17.6$ ns/km

What about for a few mode case?

$$b_i = \frac{\bar{n}_i - n_2}{n_1 - n_2}$$

$$\bar{n}_i = n_2 + (n_1 - n_2) b_i$$

$$\sin\theta_i = \frac{\bar{n}_i}{n_1} = \frac{n_2}{n_1} + \left(1 - \frac{n_2}{n_1}\right) b_i$$

$$V_{gi} = \frac{c}{n_{1g}} \left(\frac{n_2}{n_1} + \left(1 - \frac{n_2}{n_1}\right) b_i \right)$$

$$= \frac{c}{n_{1g}} \left[\frac{n_2}{n_1} + \frac{n_1 - n_2}{n_1} b_i \right]$$

$$= \frac{c}{n_{1g}} \left[\frac{n_2}{n_1} + \Delta b_i \right]$$

$$V_{gi} \approx \frac{c}{n_{1g}} (1 + \Delta b_i)$$

What about right after cutoff?

$b_1 = 0.6$, $b_2 = 0$

$D_{inter} = 10.6$ ns/km

very close to the worst case of 17.6 ns/km

For graded index optical fiber the dispersion is

$$D_{inter} = \frac{n_{1g}}{c} \frac{\Delta^2}{8}$$

for standard fiber $D_{inter} = 248$ ps/km much lower