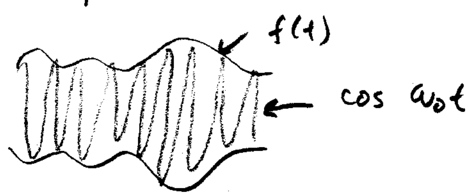
Dispersion

Now that have calculated the modes supported by an optical fiber, we need to see how the modes vary with propagation. How is the signal degraded with propagation?

Let's start by looking at a general modulated traveling wave.

$$p(t) = f(t)\cos\omega_0 t = \mathrm{Re}\{f(t)\, e^{j\omega_0 t}\}$$

$f(t)$ is a relatively narrow band and is slowly varying with respect to the carrier (ω_0).



let $s(t) = f(t)\, e^{j\omega_0 t}$ so $p(t) = \mathrm{Re}\{s(t)\}$

look at the frequency components of $s(t)$

$$S(\omega) = \int_{-\infty}^{\infty} s(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{j\omega_0 t}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{-j(\omega-\omega_0)t}\, dt$$

$$= F(\omega-\omega_0) \qquad \text{where} \qquad F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

Remember that we are in the phasor domain!

As we travel in the z-direction the form of the phasor is

$$S_0(\omega) = \underset{\substack{\uparrow \\ \text{term allowing for attenuation}}}{A(z)}\; F(\omega-\omega_0)\; e^{-j\beta z}$$

$e^{-j\beta z}$ ← traveling wave term

Now convert back to time domain to see what the wave looks like at a particular position.

$$s_0(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_0(\omega)\, e^{+j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A(z)\, F(\omega-\omega_0)\, e^{-j\beta z}\, e^{j\omega t}\, d\omega$$

$$s_0(t) = \frac{A(z)}{2\pi}\int_{-\infty}^{\infty} F(\omega-\omega_0)\, e^{j(\omega t-\beta z)}\, d\omega$$

remember that β depends on ω

So this is hard to solve

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Since the signal is narrow band
$F(\omega-\omega_0)=0$ except around small $(\omega-\omega_0)$

This means that a Taylor series expansion is valid so

$$\beta(\omega) = \beta(\omega_0) + (\omega-\omega_0)\left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_0} + \frac{(\omega-\omega_0)^2}{2}\left.\frac{d^2\beta}{d\omega^2}\right|_{\omega=\omega_0} + \cdots$$

$$\beta \approx \beta_0 + (\omega-\omega_0)\beta_0'$$

use this approximation to solve for $S_0(t)$

$$S_0(t) = \frac{A(z)}{2\pi}\int_{-\infty}^{\infty} F(\omega-\omega_0)\, e^{j(\omega t-\beta z)}\, d\omega$$

$$= \frac{A}{2\pi}\int_{-\infty}^{\infty} F(\omega-\omega_0)\, e^{j(\omega t-\beta_0 z-(\omega-\omega_0)\beta_0' z)}\, d\omega$$

$$= \frac{A}{2\pi}\int_{-\infty}^{\infty} F(\omega-\omega_0)\, \exp\left[j\left((\omega-\omega_0)t+\omega_0 t-\beta_0 z-(\omega-\omega_0)\beta_0' z\right)\right] d\omega$$

let $u=\omega-\omega_0$

$$= \frac{A}{2\pi}\int_{-\infty}^{\infty} F(u)\, \exp\left[j\left(ut-u\beta_0' z+\omega_0 t-\beta_0 z\right)\right] du$$

$$= \frac{A}{2\pi} e^{j(\omega_0 t-\beta_0 z)}\int_{-\infty}^{\infty} F(u)\, e^{ju(t-\beta_0' z)}\, du$$

this produces a time shift

$$S_0(t) = \frac{A}{2\pi} e^{j(\omega_0 t-\beta_0 z)}\, f(t-\beta_0' z)$$

Now convert from phasor domain to time domain

$$p_0(t) = \mathrm{Re}\{S_0(t)\}$$

$$= \frac{A}{2\pi} f(t-\beta_0' z)\cos(\omega_0 t-\beta_0 z)$$

We end up with an envelope and a carrier. Each one has a different traveling speed.

Let's look at the carrier first.

The phase stays the same when $\omega_0 t-\beta_0 z=0$

$$\beta_0 z = \omega_0 t$$

$$z = \frac{\omega_0}{\beta_0}t$$

$$\frac{\partial z}{\partial t} = \frac{\omega_0}{\beta_0}$$

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So the speed of the phase front is

$$\boxed{V_c = \frac{\omega_0}{\beta_0}}$$

Now look at the speed of the envelope

$$\beta_0' z = t$$

$$z = \frac{t}{\beta_0'}$$

$$\frac{dz}{dt} = V_{env} = \frac{1}{\beta_0'} = \left(\frac{\partial \beta}{\partial \omega}\right)^{-1}$$

So the envelope speed is $\boxed{V_g = \left(\frac{\partial \beta}{\partial \omega}\right)^{-1}}$

This is the speed of a pulse. This is the speed that we care about.